Classical moduli hair for Kerr black holes in String Gravity

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Abstract

We compute the external moduli and dilaton hair to $O(\alpha')$ in the framework of the one-loop corrected superstring effective action for a rotating black hole background.

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Superstring [1] theory is our best existing candidate for a consistent quantum theory of gravity which also has the prospect of unification with all other interactions.Einstein's theory which has been very successful as a classical theory of gravitation is incorporated in this more general framework.However,Superstrings involve a characteristic length of the order of the Planck scale and are expected to lead to drastic modifications of the Einstein Action at short distances.These modifications arise either due to the contribution of the infinite tower of massive string modes,appearing as α' corrections,or due to quantum loop effects.An effective low energy Lagrangian [2] that incorporates the above, involving only the massless string modes,can be derived from string theory using a perturbative approach in both the string tension α' and the string coupling.The relevant massless fields,apart from the graviton and other gauge fields,are the dilaton,which plays the role of the field-dependent string coupling that parametrizes the string loop expansion, and the moduli fields that describe the size and the shape of the internal compactification manifold.

In Einstein gravity,minimally coupled to other fields,the most general black hole solution is described by the Kerr-Newman family of rotating charged black hole solutions [3].In agreement with the "no-hair"theorem [4] at the classical level the only external fields present are those required by gauge invariance.A qualitative new feature present in the superstring effective action is the appearance of external field strength hair for the axion and dilaton fields [5] [6] [7] [8] [9] [10] [11].The tree level effective action has been calculated up to several orders in the α'-expansion.It turns out that there is no dependence on the moduli fields at tree level.The one-loop corrections to gravitational and gauge couplings have been calculated in the context of orbifold compactifications of the heterotic superstring [12].It has been shown that there are no moduli-dependent corrections to the Einstein term while there are non trivial $\mathcal{R}^2$-contributions appearing in a Gauss-Bonnet combination multiplied by a moduli-dependent coefficient function.This term is subject to a non-renormalization theorem which implies that all higher-loop moduli-dependent $\mathcal{R}^2$-contributions vanish.It is interesting to note the existence of singularity-free [13] solutions of the field equations in a Friedmann-Robertson-Walker background depending crucially on the presence of the Gauss-Bonnet term.

In the present short article we extend existing treatments [5] – [11] of black hole solutions in string gravity to iclude moduli fields.For simplicity we restrict ourselves to the zero charge case although the case of charged black holes is expected not to lead to any extra complication. Our action is the low energy effective action derived in the context of orbifold compactifications of the heterotic superstring to one-loop and α'-order.We compute to α'-order the moduli and dilaton-hair together with the corresponding two axions hair.The result although expected from previous existing investigations without the moduli fields serves to establish even better the qualitatively new features of string gravity in contrast to Einstein gravity characterized by the "no-hair" theorem.In our action we have not introduced any potential for the above fields although it is likely that in the full quantum string theory such a potential and a (small) mass is generated through non-perturbative effects.Nevertheless,if the black hole size,or the distance from the black hole ,is small compared to their inverse mass the solutions found are valid to

a good approximation.

Let us consider the universal part of the effective action of any four-dimensional heterotic superstring model which describes the dynamics of the graviton,the dilaton S and,for simplicity,the common modulus field T.At the tree level and up to first order in α' it takes the form

$$S^{(o)}_{eff} = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + \frac{|DS|^2}{(S+\overline{S})^2} + 3\frac{|DT|^2}{(T+\overline{T})^2} + \frac{\alpha'}{8}(ReS)\mathcal{R}^2_{GB} + \frac{\alpha'}{8}(ImS)\mathcal{R}\tilde{\mathcal{R}}\right) \quad (1)$$

where

$$\mathcal{R}^2_{GB} \equiv R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \quad (2)$$

and

$$\mathcal{R}\tilde{\mathcal{R}} \equiv \eta^{\mu\nu\rho\sigma}R^{\kappa\lambda}{}_{\mu\nu}R_{\rho\sigma\kappa\lambda} \quad (3)$$

Note that[1] $\eta^{\mu\nu\rho\sigma} \equiv \epsilon^{\mu\nu\rho\sigma}(-g)^{-1/2}$. We have chosen units such that $k \equiv \sqrt{8\pi G_N} \equiv 1$.

The one-loop corrections give a modulus dependence to the quadratic gravitational terms that are of the form

$$S^{(1)}_{eff} = \int d^4x\sqrt{-g}\left(\alpha'\Delta(T,\overline{T})\mathcal{R}^2_{GB} + \alpha'\Theta(T,\overline{T})\mathcal{R}\tilde{\mathcal{R}}\right) \quad (4)$$

The functions $\Delta(T,\overline{T})$ and $\Theta(T,\overline{T})$ have been derived in ref.[12] and depend multiplicatively through a coefficient on the supermultiplet content of the string model.Indroducing the notation

$$S \equiv (e^{\phi} + ia)/g^2 \quad , \quad T \equiv e^{\sigma} + ib \quad (5)$$

and reffering to ϕ as the dilaton, to σ as the modulus,to a and b as the axions and to g^2 as the string coupling ,we can write the effective one-loop, $O(\alpha')$ Langrangian as

$$\begin{aligned} \mathcal{L}_{eff} &= \frac{1}{2}R + \frac{1}{4}(\partial_\mu\phi)^2 + \frac{1}{4}e^{-2\phi}(\partial_\mu a)^2 + \frac{3}{4}(\partial_\mu\sigma)^2 + \frac{3}{4}e^{-2\sigma}(\partial_\mu b)^2 \\ &+ \alpha'\left(\frac{e^{\phi}}{8g^2} + \Delta\right)\mathcal{R}^2_{GB} + \alpha'\left(\frac{a}{8g^2} + \Theta\right)\mathcal{R}\tilde{\mathcal{R}} \end{aligned} \quad (6)$$

The equations of motion resulting from (6) are four equations for the scalar and pseudoscalar fields

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}\partial^\mu\phi] = -e^{-2\phi}(\partial_\mu a)^2 + \frac{\alpha'}{4g^2}e^{\phi}\mathcal{R}^2_{GB} \quad (7)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}e^{-2\phi}\partial^\mu a] = \frac{\alpha'}{4g^2}\mathcal{R}\tilde{\mathcal{R}} \quad (8)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}\partial^\mu\sigma] = -e^{-2\sigma}(\partial_\mu b)^2 + \frac{2\alpha'}{3}\left(\frac{\delta\Delta}{\delta\sigma}\right)\mathcal{R}^2_{GB} + \frac{2\alpha'}{3}\left(\frac{\delta\Theta}{\delta\sigma}\right)\mathcal{R}\tilde{\mathcal{R}} \quad (9)$$

[1] $\epsilon^{oijk} = -\epsilon_{ijk}$

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}e^{-2\sigma}\partial^\mu b] = \frac{2\alpha'}{3}\left(\frac{\delta\Delta}{\delta b}\right)\mathcal{R}^2_{GB} + \frac{2\alpha'}{3}\left(\frac{\delta\Theta}{\delta b}\right)\mathcal{R}\tilde{\mathcal{R}} \tag{10}$$

and the equation[2]

$$\begin{aligned}
&R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \alpha'(g_{\mu\rho}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\rho})\eta^{\kappa\lambda\alpha\beta}D_\gamma(\tilde{R}^{\rho\gamma}{}_{\alpha\beta}D_\kappa f_1) - 8\alpha' D_\rho(\tilde{R}_\mu{}^\lambda{}_\nu{}^\rho D_\lambda f_2) = \\
&-\frac{1}{2}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{1}{4}g_{\mu\nu}(\partial_\rho\phi)^2 - \frac{e^{-2\phi}}{2}(\partial_\mu a)(\partial_\nu a) + \frac{1}{4}g_{\mu\nu}e^{-2\phi}(\partial_\rho a)^2 \\
&-\frac{3}{2}(\partial_\mu\sigma)(\partial_\nu\sigma) + \frac{3}{4}g_{\mu\nu}(\partial_\rho\sigma)^2 - \frac{3}{2}e^{-2\sigma}(\partial_\mu b)(\partial_\nu b) + \frac{3}{4}e^{-2\sigma}g_{\mu\nu}(\partial_\rho b)^2
\end{aligned} \tag{11}$$

We have introduced the functions

$$f_1 \equiv \frac{e^\phi}{8g^2} + \Delta \quad , \quad f_2 \equiv \frac{a}{8g^2} + \Theta \tag{12}$$

At this point we introduce the Kerr metric anticipating that it will continue to be a solution to $O(\alpha')$

$$ds^2 = \left(\frac{\rho^2 - 2Mr}{\rho^2}\right)dt^2 - \frac{\rho^2}{\Lambda}dr^2 - \rho^2 d\theta^2 + \frac{4MrAsin^2\theta}{\rho^2}dtd\varphi - \frac{sin^2\theta}{\rho^2}\Sigma^2 d\varphi^2 \tag{13}$$

where $\rho^2 \equiv r^2 + A^2cos^2\theta, \Lambda \equiv r^2 + A^2 - 2Mr$ and $\Sigma^2 \equiv (r^2 + A^2)^2 - \Lambda A^2 sin^2\theta. A$ stands for the angular momentum per unit mass.For this metric we can calculate

$$\mathcal{R}\tilde{\mathcal{R}} = \frac{192M^2Arcos\theta(3r^2 - A^2cos^2\theta)(r^2 - 3A^2cos^2\theta)}{(r^2 + A^2cos^2\theta)^6} \tag{14}$$

$$\mathcal{R}^2_{GB} = \frac{48M^2(r^2 - A^2cos^2\theta)[(r^2 + A^2cos^2\theta)^2 - 16r^2A^2cos^2\theta]}{(r^2 + A^2cos^2\theta)^6} \tag{15}$$

Since,as we declared in the introduction,we plan to determine solutions to $O(\alpha')$ let us first obtain the zeroth order solutions for the scalar and pseudoscalar fields.Introducing a rescaled axion field $\partial_\mu\tilde{a} \equiv e^{-2\phi}\partial_\mu a$ we can write the dilatonic-axion equation of motion in the form

$$\frac{\partial}{\partial r}\left[(r^2 - 2Mr + A^2)\frac{\partial\tilde{a}}{\partial r}\right] + \frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left[sin\theta\frac{\partial\tilde{a}}{\partial\theta}\right] = 0 \tag{16}$$

It has a general solution of the form

$$\tilde{a} = \sum_{l=0}^{\infty} P_l(cos\theta)\left[A_lQ_l(z) + B_lP_l(z)\right] \tag{17}$$

where $z \equiv (r - M)/\sqrt{M^2 - A^2}$. Imposing the black hole boundary condition[3] $r \to r_H$ or $z \to 1$ forces us to require $A_l = 0$, $\forall l$.On the other hand requiring finiteness at $r \to \infty$

[2] $\tilde{R}^{\mu\nu}{}_{\kappa\lambda} = \eta^{\mu\nu\rho\sigma}R_{\rho\sigma\kappa\lambda}$
[3] $r_H = M + \sqrt{M^2 - A^2}$

or $z \to \infty$ forces us to set $B_l = 0$, $\forall l \geq 1$. Thus, only the constant solution $\tilde{a} = B_o$ is possible.Using that,the dilaton equation reduces,to zeroth order,to the form $D^2\phi = 0$ which for the same reasons as in the case of the axion $\tilde{a}$ leads us to the conclusion that to this order the dilaton is a constant. Following the same procedure for the modulus and its associated axion we also arrive at constant zeroth order values.

In order to proceed and obtain the $O(\alpha')$ solutions we need the static axisymmetric Green's function defined by the equation

$$\frac{1}{\sqrt{-g}}\partial_\mu \left[\sqrt{-g}g^{\mu\nu}\partial_\nu G(x-y)\right] = \frac{\delta^{(3)}(x-y)}{\sqrt{-g}} \tag{18}$$

which for our metric (13) becomes

$$\frac{\partial}{\partial r}\left[(r^2 + A^2 - 2Mr)\frac{\partial G}{\partial r}\right] + \frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left[sin\theta\frac{\partial G}{\partial\theta}\right] = -\delta(r-r_o)\delta(cos\theta - cos\theta_o)\delta(\varphi-\varphi_o) \tag{19}$$

for a point source located at r_o , θ_o , φ_o.Demanding finiteness at $r = r_H$ and at infinity we obtain

$$G(r,\theta,\varphi;r_o,\theta_o,\varphi_o) = \sum_{l=0}^{\infty} R_l(r,r_o)P_l(cos\gamma) \tag{20}$$

with

$$cos\gamma \equiv cos\theta cos\theta_o + sin\theta sin\theta_o cos(\varphi-\varphi_o) \tag{21}$$

and

$$\begin{aligned} R_l(r,r_o) &= -\frac{(2l+1)}{4\pi\sqrt{M^2-A^2}}\left[P_l\left(\frac{(r_o-M)}{\sqrt{M^2-A^2}}\right)Q_l\left(\frac{(r-M)}{\sqrt{M^2-A^2}}\right)\theta(r-r_o)\right. \\ &+ \left. P_l\left(\frac{(r-M)}{\sqrt{M^2-A^2}}\right)Q_l\left(\frac{(r_o-M)}{\sqrt{M^2-A^2}}\right)\theta(r_o-r)\right] \end{aligned} \tag{22}$$

Using the Green's function we can write the external dilaton solution as

$$\phi(r,\theta,\varphi) = \int_{r_H}^{\infty} dr_o \int_{-1}^{1} dcos\theta_o \int_0^{2\pi} d\varphi_o (r_o^2 + A^2 cos^2\theta_o) G(r,\theta,\varphi;r_o,\theta_o,\varphi_o)\mathcal{J}(r_o,\theta_o,\varphi_o) \tag{23}$$

where the source $\mathcal{J}$ is the right hand side of equation (7).Similar expressions hold for the rest of the scalar and pseudoscalar fields[4] σ,$\tilde{a}$,$\tilde{b}$ in terms of the corresponding source terms.It is straightforward but tedious to obtain the $O(\alpha')$ expressions for these fields.At the same time ,since all scalar and pseudoscalar fields have non-constant parts of order α',the right hand side of equation (11) is $O(\alpha'^2)$ and thus the gravitational part of the solution is the standard Kerr metric (13).The $O(\alpha')$ fields are

[4] $\partial_\mu \tilde{b} = e^{-2\sigma}\partial_\mu b$

$$
\begin{aligned}
\phi(r,\theta) &= \phi_o - \frac{\alpha' e^{\phi_o}}{g^2}\left[\frac{1}{A^2} ln\left(\frac{r-M+\sqrt{M^2-A^2}}{\sqrt{A^2+r^2}}\right) + \frac{Mr}{(A^2+r^2)^2}\right.\\
&+ \left.\frac{(2A^2-M^2)}{2A^3M}\left(\frac{\pi}{2} - Arctan(\frac{r}{A})\right) + \frac{2A^2+Mr}{2A^2(A^2+r^2)}\right] P_o(cos\theta) + ... \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{a}(r,\theta) &= \tilde{a}_o - \frac{6\alpha' A}{g^2(M^2-A^2)}\left\{(r-M)\left[\frac{(A^2-M^2)}{A^4} ln\left(\frac{r-M+\sqrt{M^2-A^2}}{\sqrt{A^2+r^2}}\right)\right.\right.\\
&+ \left.\frac{2A^2+Mr-M^2}{2A^2(A^2+r^2)} + \frac{(A^2-M^2)}{A^3M}\left(\frac{\pi}{2} - Arctan(\frac{r}{A})\right)\right] + \frac{A^2\sqrt{M^2-A^2}}{Mr_H^3}\\
&+ \left.\frac{(3M-4r_H)}{2r_H^2} + \frac{M(M^2+r^2)}{(A^2+r^2)^2}\right\} P_1(cos\theta) + ... \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
\sigma(r,\theta) &= \sigma_o - \frac{8\alpha'}{3}\left(\frac{\partial\Delta}{\partial\sigma}\right)_{\sigma_o,b_o}\left[\frac{1}{A^2} ln\left(\frac{r-M+\sqrt{M^2-A^2}}{\sqrt{A^2+r^2}}\right) + \frac{Mr}{(A^2+r^2)^2}\right.\\
&+ \left.\frac{(2A^2-M^2)}{2A^3M}\left(\frac{\pi}{2} - Arctan(\frac{r}{A})\right) + \frac{2A^2+Mr}{2A^2(A^2+r^2)}\right] P_o(cos\theta)\\
&- \frac{2\alpha'}{3}\left(\frac{\partial\Theta}{\partial\sigma}\right)_{\sigma_o,b_o}\frac{24A}{(M^2-A^2)}\left\{(r-M)\left[\frac{(A^2-M^2)}{A^4} ln\left(\frac{r-M+\sqrt{M^2-A^2}}{\sqrt{A^2+r^2}}\right)\right.\right.\\
&+ \left.\frac{2A^2+Mr-M^2}{2A^2(A^2+r^2)} + \frac{(A^2-M^2)}{A^3M}\left(\frac{\pi}{2} - Arctan(\frac{r}{A})\right)\right] + \frac{A^2\sqrt{M^2-A^2}}{Mr_H^3}\\
&+ \left.\frac{(3M-4r_H)}{2r_H^2} + \frac{M(M^2+r^2)}{(A^2+r^2)^2}\right\} P_1(cos\theta) + ... \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{b}(r,\theta) &= \tilde{b}_o - \frac{8\alpha'}{3}\left(\frac{\partial\Delta}{\partial b}\right)_{\sigma_o,b_o}\left[\frac{1}{A^2} ln\left(\frac{r-M+\sqrt{M^2-A^2}}{\sqrt{A^2+r^2}}\right) + \frac{Mr}{(A^2+r^2)^2}\right.\\
&+ \left.\frac{(2A^2-M^2)}{2A^3M}\left(\frac{\pi}{2} - Arctan(\frac{r}{A})\right) + \frac{2A^2+Mr}{2A^2(A^2+r^2)}\right] P_o(cos\theta)\\
&- \frac{2\alpha'}{3}\left(\frac{\partial\Theta}{\partial b}\right)_{\sigma_o,b_o}\frac{24A}{(M^2-A^2)}\left\{(r-M)\left[\frac{(A^2-M^2)}{A^4} ln\left(\frac{r-M+\sqrt{M^2-A^2}}{\sqrt{A^2+r^2}}\right)\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{2A^2+Mr-M^2}{2A^2(A^2+r^2)} + \frac{(A^2-M^2)}{A^3M}\left(\frac{\pi}{2} - Arctan(\frac{r}{A})\right)\Bigg] + \frac{A^2\sqrt{M^2-A^2}}{Mr_H^3} \\
&+ \quad \frac{(3M-4r_H)}{2r_H^2} + \frac{M(M^2+r^2)}{(A^2+r^2)^2}\Bigg\} P_1(cos\theta) + ... \qquad (27)
\end{aligned}
$$

The leading modulus and $\tilde{b}$-hair behaviour is that of a monopole term analogous to the dilaton.This is evident from the slow rotation limit of the dilaton solution

$$
\begin{aligned}
\phi(r,\theta) &= \phi_o + \frac{\alpha' e^{\phi_o}}{4g^2}\left[-\frac{2}{Mr}(1+\frac{M}{r}+\frac{4M^2}{3r^2}) + \frac{A^2}{2M^3r}(\frac{1}{2}+\frac{M}{r}+\frac{12M^2}{3r^2}\right. \\
&+ \left.\frac{6M^3}{r^3}+\frac{64M^4}{5r^4}) + ...\right] P_o(cos\theta) + ... \qquad (28)
\end{aligned}
$$

Note that the coefficient functions[12] Δ and Θ are such that they have an extremum at the self-dual point $\sigma_o = \tilde{b}_o = 0$.Perturbing around the self-dual solution leads to vanishing modulus hair to $O(\alpha')$.The infinite continuum of non-zero σ_o,$\tilde{b}_o$ values allows for the non-vanishing modulus and $\tilde{b}$-axion hair given by (26) and (27).

Although the existence of non-trivial dilaton,moduli and axion fields outside a Kerr black hole seems to violate the letter of the “no-hair theorem” it does not violate the spirit since the solution is uniquelly characterized by mass and angular momentum.In the terminology introduced by S.Coleman,J.Preskill and F.Wilczek [14] the external moduli and dilaton hair are examples of “secondary ” hair.

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References

[1] For a review,see :M.Green , J.Schwarz and E.Witten, “Superstring Theory ” (Cambridge U.P., Cambridge,1987).

[2] C.G.Callan, D.Friedan, E.J.Martinec and M.J.Perry, Nucl.Phys. B262 (1985)543 ; B278 (1986)78.
E.S.Fradkin and A.A.Tseytlin, Phys.Lett.B158(1985)316; Nucl.Phys.B262(1985)1
A.Sen,Phys.Rev. D32 (1985)2142; Phys.Rev.Lett.55(1985)1846.

[3] R.M.Wald ,General Relativity (University of Chicago Press, Chicago,1984) and references therein.

[4] W.Israel,Phys.Rev.164(1967)331;for a review,
in “General Relativity, an Einstein Centennary Survey” edit.S.W.Hawking and W.Israel(Cambridge U.P.,Cambridge,1979).
B.Carter,Phys.Rev.Lett.26(1971)331
R.Price,Phys.Rev.D5(1972)2419,2439
J.D.Bekenstein,Phys.Rev.D5(1972)1239,2403
C.Teitelboim,Phys.Rev.D5(1972)2941
D.C.Robinson,Phys.Rev.Lett.34(1975)405
R.M.Wald,Phys.Rev.Lett.26(1971)1653
E.D.Fackerell and J.R.Ipser,Phys.Rev.D5(1972)2455
S.L.Adler and R.B.Pearson,Phys.Rev.D18(1978)2798

[5] G.W.Gibbons,Nucl.Phys.207(1982)337
G.W.Gibbons and K.Maeda,Nucl.Phys.B298(1988)741

[6] C.G.Callan,R.C.Myers and M.J.Perry,Nucl.Phys.B311(1988/89)673

[7] B.Campbell,M.Duncan,N.Kaloper and K.A.Olive,Phys.Lett.B251(1990)34

[8] D.Garfinkle,G.T.Horowitz and A.Strominger,Phys.Rev.D43(1991)3140

[9] B.Campbell,N.Kaloper and K.A.Olive,Phys.Lett.B263(1991)364

[10] A.Shapere,S.Trivedi and F.Wilczek,Mod.Phys.Lett.A6(1991)2677

[11] B.Campbell,N.Kaloper and K.A.Olive,Phys.Lett.B285(1992)199

[12] I.Antoniadis,E.Gava and K.S.Narain,Phys.Lett.B283(1992)209;
Nucl.Phys.B393(1992)93

[13] I.Antoniadis,J.Rizos and K.Tamvakis,Nucl.Phys.B415(1994)497

[14] S.Coleman,J.Preskill and F.Wilczek,Nucl.Phys.B378(1992)175